Suggested solutions to the IO (BSc) exam on June 11, 2013
VERSION: July 2, 2013

Question 1a)

From the lecture slides (but I have cut some bits that are not needed to answer this question):

- We can conclude that firm 1's full information demand equals

$$\boxed{Q_1^{FI}(p_1,p_2) = \tfrac{p_2-p_1+\tau}{2\tau}.}$$

- Its actual demand, given advertising levels λ_1 and λ_2, equals

$$\boxed{Q_1(p_1,p_2) = \lambda_1\left[(1-\lambda_2)+\lambda_2\left(\tfrac{p_2-p_1+\tau}{2\tau}\right)\right].}$$

- Firm 1's maximization problem:

$$\boxed{\max_{p_1,\lambda_1}\left\{\lambda_1\left[(1-\lambda_2)+\lambda_2\left(\tfrac{p_2-p_1+\tau}{2\tau}\right)\right](p_1-c)-\tfrac{a}{2}\lambda_1^2\right\}.}$$

- FOC w.r.t. p_1:

$$\boxed{-\left(\tfrac{\lambda_1\lambda_2}{2\tau}\right)(p_1-c)+\lambda_1\left[(1-\lambda_2)+\lambda_2\left(\tfrac{p_2-p_1+\tau}{2\tau}\right)\right]=0}\Leftrightarrow$$

$$\boxed{\text{Best reply: } p_1 = \underbrace{\frac{c+p_2+\tau}{2}}_{\text{Same as w full info}} + \underbrace{\left(\frac{1-\lambda_2}{\lambda_2}\right)\tau}_{\text{new term}}}$$

- FOC w.r.t. λ_1:

$$\boxed{\underbrace{a\lambda_1}_{\text{MC of adv.}} = \underbrace{\overbrace{\left[(1-\lambda_2)+\lambda_2\left(\frac{p_2-p_1+\tau}{2\tau}\right)\right]}^{\text{Impact on exp. sales of raising }\lambda_1\text{ 1 unit}}\overbrace{(p_1-c)}^{\text{Price-MC margin}}}_{\text{MB of adv:}}}\Leftrightarrow$$

$$\boxed{\text{Best reply: } \lambda_1 = \tfrac{1}{a}\left[(1-\lambda_2)+\lambda_2\left(\tfrac{p_2-p_1+\tau}{2\tau}\right)\right](p_1-c)}$$

- Restate p_1-FOC, impose symmetry and solve for p:

$$p_1 = \frac{c+p_2+\tau}{2}+\left(\frac{1-\lambda_2}{\lambda_2}\right)\tau \Rightarrow \boxed{p^* = c+\left(\tfrac{2-\lambda^*}{\lambda^*}\right)\tau.}$$

- Restate λ_1-FOC, impose symmetry and solve for λ:

$$a\lambda_1 = \left[(1-\lambda_2)+\lambda_2\left(\frac{p_2-p_1+\tau}{2\tau}\right)\right](p_1-c)\Leftrightarrow$$

$$a\lambda^* = \left[1 - \frac{\lambda^*}{2}\right](p^* - c) = \left[\frac{2-\lambda^*}{2}\right]\overbrace{\left(\frac{2-\lambda^*}{\lambda^*}\right)\tau}^{=p^*-c} \Leftrightarrow$$

$$\frac{2a}{\tau}(\lambda^*)^2 = (2-\lambda^*)^2 \Leftrightarrow \pm\sqrt{\frac{2a}{\tau}}(\lambda^*) = (2-\lambda^*)$$

$$\Rightarrow \boxed{\lambda^* = \frac{2}{1+\sqrt{\frac{2a}{\tau}}}} \text{ and } \boxed{p^* = c + \left(\frac{2-\lambda^*}{\lambda^*}\right)\tau = c + \sqrt{2a\tau}.}$$

Question 1b)

- Quote from Tirole, page 293 [note that he uses the notation Φ for the variable that Belleflamme & Peitz call λ]:

  What is more remarkable, [equilibrium profits] increase with the cost of advertising. The direct effect of an increase in a (for p and Φ given) is to reduce the firms' profits. However, there is a strategic effect: An increase in advertising costs reduces advertising and thus increases informational product differentiation. This allows the firms to raise the price. In this example, they gain more from costlier advertising than they lose. This result is not general, but it strongly exemplifies the role of advertising in reducing product differentiation. It may also shed some light on why some professions do not resist—and sometimes encourage—legal restrictions om advertising.

Question 1c)

- (i) Kreps and Scheinkman studied a two-stage game where the firms, in the first stage, simultaneously choose capacities $\overline{q}_i$ (at some cost). Then at stage 2, knowing each other's capacity, the firms simultaneously choose prices p_i.

- (ii) The result that they could show can be summarized as follows:

  – Suppose the demand function is concave and the rationing rule is the efficient one.

  – Then the outcome (i.e., the equilibrium capacities/quantities and the equilibrium price) of the two-stage game is the same as that of the corresponding one-stage Cournot game.

- (iii) The result is a celebrated one and many economists interpret it as a justification for thinking of Cournot games as a reduced form representation of the two-stage game described above. This is appealing, because the story in the two-stage game sounds plausible and realistic (in particular, in that story there is someone who actually sets the prices, in contrast to the Cournot model). At the same time, the outcome is not as unrealistic as in the Bertrand model, where the equilibrium involves marginal cost pricing even when there are only two firms. So the outcome of the two-stage game combines the good and appealing features of the Bertrand and Cournot models, while avoiding the drawbacks with each of those models. However, there are some caveats:

- The result obtained by Kreps and Scheinkman, which can be referred to as "*Cournot outcome in the two-stage game*", is **weaker** than our result under a) where we obtained the "*exact Cournot reduced form*". With the latter result, we actually get exactly the Cournot profit functions, $\pi_i = [1 - (\overline{q}_1 + \overline{q}_2)]\,\overline{q}_i - c_0\overline{q}_i$ (where c_0 is the investment cost). This means that we in that case can also study a version of the Cournot model with, for example, sequential quantity choices.
- The Kreps-Scheinkman result is not very robust to changes in the assumptions. For example, it relies critically on the assumption of the efficient-rationing rule.
- In more general settings, the capacity choices in the full game may serve important roles that are not captured by a reduced form. For example, firms with **private information** may want to use the capacity choices as **informative signals** to its rivals.

• A summary of Tirole's discussion of the implications of Kreps-Scheinkman's result:

- The predictions and welfare results of the traditional Cournot model can be provided with foundations in some extreme cases.
- The two-stage game illustrates a broad idea that firms may want to choose non-price actions that soften price competition.
- In many applications the exact Cournot profit functions are not essential. Instead the key thing is that the best-response functions are downward-sloping—i.e., that the firms' choice variables are **strategic substitutes**:

$$\boxed{\frac{\partial^2 \pi_i}{\partial \overline{q}_i \partial \overline{q}_j} = \frac{\partial^2\left(\left[P\left(\overline{q}_i+\overline{q}_j\right)-c_0\right]\overline{q}_i\right)}{\partial \overline{q}_i \partial \overline{q}_j} = P' + P''\overline{q}_i < 0.}$$

This may very well hold even if the "*exact Cournot reduced form*" does not hold (Kreps-Sheinkman assumed $P'' \leq 0$).

Question 2a)

- The firm's profits can be written as

$$\pi = p_o n_o + p_y n_y.$$

The demand expressions (n_o and n_y) are given by (1) and (2) in the question (under Assumption 1, the middle lines are the relevant ones). We therefore get

$$\pi = \gamma p_o (1 - p_o) + (1 - \gamma) p_y (1 + \nu\gamma - p_y - \nu\gamma p_o).$$

- The firm wants to maximize these profits w.r.t. p_o and p_y. The second-order condition appears to be unproblematic. (The second derivatives $\partial^2\pi/\partial p_o^2$ and $\partial^2\pi/\partial p_y^2$ are clearly negative. This speaks in favor of the second-order condition being satisfied. To check this carefully by investigating also the cross derivative and the Hessian is not required.) Also, by Assumption 1 we know that the optimal prices are in the range where the middle lines of the demand functions are the relevant ones. We can therefore characterize the optimal prices with the help of the first-order conditions.

- The FOC w.r.t. p_o is:

$$\frac{\partial \pi}{\partial p_o} = \gamma (1 - 2p_o) - (1 - \gamma) \nu\gamma p_y = 0$$

or

$$2p_o + \nu (1 - \gamma) p_y = 1.$$

- The FOC w.r.t. p_y is:

$$\frac{\partial \pi}{\partial p_y} = (1 - \gamma)(1 + \nu\gamma - 2p_y - \nu\gamma p_o) = 0$$

or

$$2p_y + \nu\gamma p_o = 1 + \nu\gamma.$$

- We can now solve the above linear equation system for p_o and p_y. Any method for doing that is fine. Here I use Cramer's rule. On matrix form the equation system becomes:

$$\underbrace{\begin{bmatrix} 2 & \nu(1-\gamma) \\ \nu\gamma & 2 \end{bmatrix}}_{=A} \begin{bmatrix} p_o \\ p_y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 + \nu\gamma, \end{bmatrix}$$

where we note that $\det(A) = 4 - \nu^2\gamma(1-\gamma)$. Cramer's rule now yields

$$p_o = \frac{2 - (1 + \nu\gamma)\nu(1 - \gamma)}{4 - \nu^2\gamma(1 - \gamma)}.$$

and

$$p_y = \frac{2(1 + \nu\gamma) - \nu\gamma}{4 - \nu^2\gamma(1-\gamma)} = \frac{2 + \nu\gamma}{4 - \nu^2\gamma(1-\gamma)}.$$

- That is, the optimal prices are

$$\boxed{\left(p_o^*, p_y^*\right) = \left(\tfrac{2-(1+\nu\gamma)\nu(1-\gamma)}{4-\nu^2\gamma(1-\gamma)}, \tfrac{2+\nu\gamma}{4-\nu^2\gamma(1-\gamma)}\right).}$$

Question 2b)

- We are asked two questions. First, is the old consumers' surplus larger or smaller with price discrimination than without? Second, is the young consumers' surplus larger or smaller with price discrimination than without?

- The assumptions of the model say that young consumers obtain a higher surplus from consumption of the good, the larger is the number of old people who consume the good. Therefore, if, for some reason, demand among old consumers goes up, then that has a positive impact on the young consumers' willingness to pay and thus on the young consumers' demand. The firm realizes this, and that is why, when price discrimination is feasible, it optimally charges a lower price to old consumers and a higher one to young consumers. (The firm loses some revenue from the old consumers when lowering their price, but this leads to more old consumers buying, which makes it possible to charge the young consumers a higher price and thus earn more money in the "young market". This more than compensates for the loss in the "old market".) Thus, as is stated in the question, $p_o^* < \overline{p}^{**} = \frac{1}{2} < p_y^*$.

- The old consumers therefore pay a lower price under price discrimination, which is good for their surplus. Moreover, for old consumers there is no externality (their utility is not affected by the number of other consumers buying the good). Therefore, the lower price is the only channel through which the old consumers are affected by price discrimination. We should thus expect the old consumers' surplus to be unambiguously larger with price discrimination than without.

- We also see that the young consumers pay a higher price under price discrimination, which is bad for their surplus. However, young consumers also benefit from a network externality: their utility is higher, the larger is the number of old consumers that buy the good. Since a larger number of old consumers will buy under price discrimination (as their price then drops), this effect of price discrimination on the young consumers' surplus is positive. All in all, there is one effect (going through the higher price) that suggests that price discrimination lowers the young consumers' surplus, and there is another effect that goes in the opposite direction. Therefore it is indeed the case (as is suggested in the question) that there are effects pointing in opposite directions and it is hard to say anything about the net effect.

Question 2c)

- The claim in the question that we are asked to prove is: If $p_o \in [0,1]$, then the fulfilled-expectations demand originating from the group of young consumers is given by

$$n_y = \begin{cases} 1-\gamma & \text{if } p_y \leq \nu\gamma(1-p_o) \\ (1-\gamma)(1+\nu\gamma - p_y - \nu\gamma p_o) & \text{if } \nu\gamma(1-p_o) \leq p_y \leq 1+\nu\gamma(1-p_o) \\ 0 & \text{if } p_y \geq 1+\nu\gamma(1-p_o). \end{cases} \tag{2}$$

  The question also defines a "fulfilled-expectations demand function" (see footnote 1): A fulfilled-expectations demand function specifies the number of young consumers who want to buy, given some prices p_y and p_o and given some beliefs n_o^e about the number of old consumers who buy. Moreover, those beliefs are correct, $n_o^e = n_o$.

- The question also gives us the demand function for the group of old consumers: if $p_o \in [0,1]$, then $n_o = \gamma(1-p_o)$.

- The net utility of a young consumer, if buying the good, equals $\theta + \nu n_o^e - p_y$. Therefore, a young consumer with taste parameter $\theta \in [0,1]$ will buy the good if

$$\theta + \nu n_o^e - p_y \geq 0 \Leftrightarrow \theta \geq p_y - \nu n_o^e \equiv \widehat{\theta}.$$

  Plugging in $n_o^e = n_o = \gamma(1-p_o)$, we have

$$\boxed{\widehat{\theta} = p_y - \nu\gamma(1-p_o).}$$

  - Suppose $\widehat{\theta} \leq 0$. This is equivalent to $p_y \leq \nu\gamma(1-p_o)$, which is the requirement for demand to be given by the first line of (2). The fact that $\widehat{\theta} \leq 0$ means that all young consumers buy; hence $n_y = 1-\gamma$.
    - Conclusion: if $p_y \leq \nu\gamma(1-p_o)$, then we have $n_y = 1-\gamma$.
  - Suppose $\widehat{\theta} \geq 1$. This is equivalent to $p_y \geq 1+\nu\gamma(1-p_o)$, which is the requirement for demand to be given by the last line of (2). The fact that $\widehat{\theta} \geq 1$ means that no young consumers buy; hence $n_y = 0$.
    - Conclusion: if $p_y \geq 1+\nu\gamma(1-p_o)$, then we have $n_y = 0$.
  - Suppose $\widehat{\theta} \in [0,1]$. This is equivalent to $\nu\gamma(1-p_o) \leq p_y \leq 1 + \nu\gamma(1-p_o)$, which is the requirement for demand to be given by the middle line of (2). The fact that $\widehat{\theta} \in [0,1]$ means that the number of young consumers who buy equals

$$\begin{aligned} n_y &= (1-\gamma)\left(1-\widehat{\theta}\right) \\ &= (1-\gamma)\left[1-p_y+\nu\gamma(1-p_o)\right] \\ &= (1-\gamma)\left[1+\nu\gamma-p_y-\nu\gamma p_o\right]. \end{aligned}$$

    - Conclusion: if $\nu\gamma(1-p_o) \leq p_y \leq 1+\nu\gamma(1-p_o)$, then we have $n_y = (1-\gamma)[1+\nu\gamma-p_y-\nu\gamma p_o]$.